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TITLE: NUCLEAR STRUCTURE CONSIDERATIONS FOR GAMMA-RAY LASERS

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NUCLEAR STRUCTURE CONSIDERATIONS FOR GAMMA-RAY LASERS

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ABSTRACT

We present initial results in our investigation of the nuclear physics issues of gamma-ray lasers. These include the questions of what is known from existing experimental data, where does one optimally search for nuclei displaying simultaneously both closely lying levels and nuclear isomerism, and which theoretical models does one employ for systematic searches for candidate nuclei and for calculation of detailed candidate level properties. We address these three questions in the present paper.

I. INTRODUCTION

One of the most promising methods for development of a gamma-ray laser involves a two-step approach that produces a significant population of a long-lived isomeric (or storage) state via a relatively slow nuclear process. This state is then depopulated rapidly via an externally-induced transition to a short-lived nuclear state so as to produce lasing. To satisfy these critera, a suitable nucleus must be identified in which an isomeric state (of sufficient lifetime to allow population and preparation in a host material) lies nearby a state that decays essentially instantaneously. Because of the limitations of radiation sources needed to drive such interlevel transfers, this spacing between nuclear states must be less than several hundred electron volts. Additionally, the spin differences between these states must be such that an interlevel transfer employing successive low-angular momentum steps can occur. Of practical importance is the existence of atomic properties that would allow nuclei existing in such an isomeric state to be separated readily. Finally, the resulting interlevel decay should populate a Mossbauer transition of low enough energy as not to damage the host material. Likewise the radiation (type or intensity) from the isomer used as a storage state must not damage the host material.

In Sec. II we summarize our initial, computerized search of a nuclear structure data library for candidate nuclei for a gamma-ray laser. In Secs III and IV we present elementary nuclear physics considerations as to (a) which nuclei one expects to possess close level spacings at low excitation energies, and (b) which nuclei one expects to display isomerism. In Sec. V, we consider the union of set (a) and one subset of (b) as a first guess as to where to search for candidate nuclei for a gamma-ray laser. In Sec. VI we describe the main theoretical models that one uses as starting points in the systematic investigations to locate isomeric states as well as to calculate their properties. Our conclusions are presented in Sec. VII.

II. SEARCH OF EXISTING DATA

As part of an effort to identify possible nuclei with certain of the above level characteristics, we completed a search of the computerized nuclear structure data library, CDRL82,[1] which is based on the 1978 Table of Isotopes[2] compilation. This library contains data (excitation energy, spin, parity) for 41000 levels although, because of the vintage of the compilation (1977), current data are often lacking.

The criterion used to perform the search was initial identification of isomeric states having lifetimes greater than or equal to a specified input value. A halflife $\geq$ 5 seconds was used for these results. After identification of such a state, the spacings of nearby levels were examined to determine which ones (if any) fell within a specified excitation energy window, ΔE. (Two ΔE values were specified: 5 keV and 1 keV.) If one (or more) levels were found that lay nearby to an isomeric state of sufficient lifetime, then the energy, spin, and parity information appropriate to such levels were printed. These results were examined further to eliminate levels where spin or parity information was lacking, or where an obvious level duplication had occurred

Figure 1 illustrates regions of the periodic table where candidate nuclei having isomeric states of lifetime > 5 seconds along with short-lived levels occurring within a spacing of 5 keV are indicated. Also shown (approximately) by the shaded areas are regions of nuclear deformation where enhanced densities of nuclear levels should occur. Table I lists the nuclei identified in this search. For the next part of the search, the spacing criterion was lowered to $\lesssim$ 1 keV. Figure 2 depicts the candidate nuclei identified. It should be noted that the number of levels originally identified as satisfying the 5-keV separation was about 1.5-2 times the number of illustrated levels. However, a significant fraction was eliminated by imposition of the conditions listed earlier. Of the nuclei appearing in Fig. 2, ^{179}Hf is of particular interest because it exhibits the closest level spacing (~200 eV) of any nucleus identified. The transition from the short-lived nuclear state involves a 1.1 MeV gamma ray, which is too energetic for the gamma-ray laser, but ideal for investigation of interlevel transfer processes.

TABLE I

NUCLEI IDENTIFIED IN THE SEARCH WITH ΔE = 5 keV

$_{31}Ga^{74}$	$_{37}Rb^{86}$
$_{41}Nb^{90}$	$_{46}Pd^{111}$
$_{47}Ag^{110}$	$_{56}Ba^{133}$
$_{63}Eu^{152}$	$_{65}Tb^{158}$
$_{71}Lu^{171}$	$_{72}Hf^{179}$
$_{74}W^{183}$	$_{82}Pb^{203}$
$_{83}Bi^{201}$	$_{95}Am^{242}$

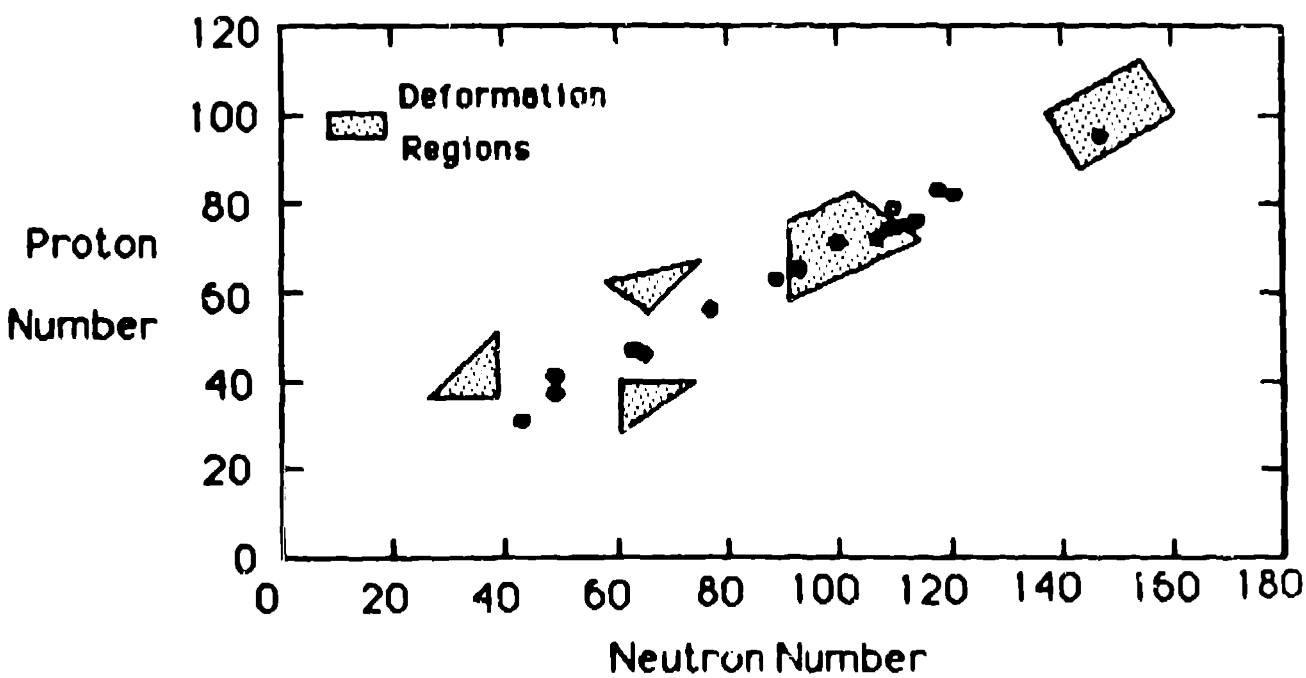

Fig. 1. Candidate nuclides with 5 keV (or less) spacings.

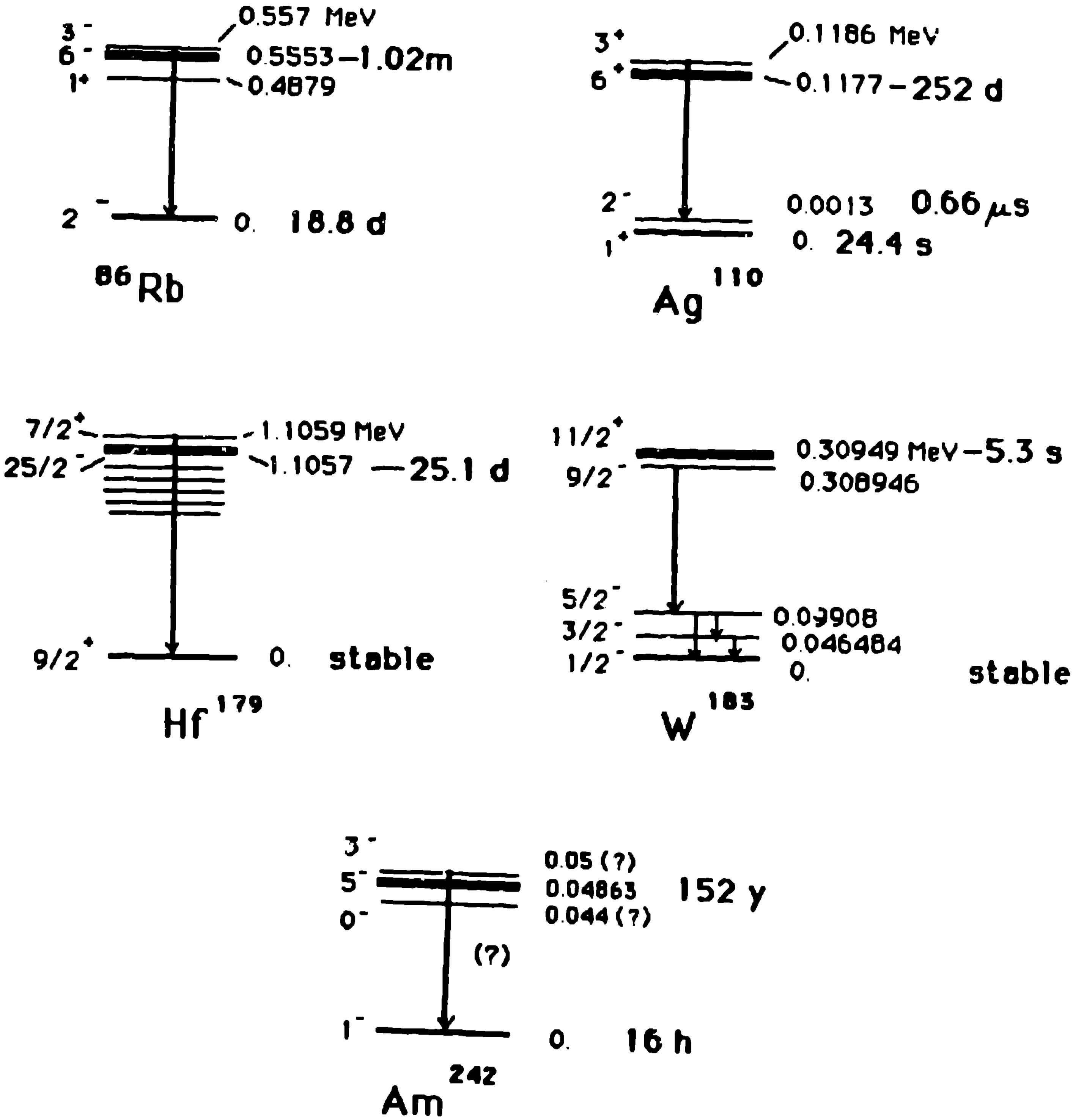

Fig. 2. Candidates having spacings ≲ 1 keV.

There are several difficulties associated with this search, most of which are related to the vintage (1977) of the compilation. Experimental techniques have progressed significantly over the past 8 years so that a more up-to-date compilation might offer many more nuclear candidates. Secondly, two data files, one resulting from nuclear decay data and the other from reaction data, were merged to produce CDRL82. In the process, levels appearing in these separate files having spacings $\lesssim$ 1 keV were considered to be the same. While this presumption is prudent for most nuclear levels, there is a significant chance that level information applicable to the gamma-ray laser could be lost. In order to circumvent certain of these problems (especially those related to use of antiquated data), a second search has begun using nuclear structure data from the more recent (circa 1982 for some nuclei) ENDSF[3] evaluated nuclear structure file.

III. CLOSE LEVEL SPACINGS IN NUCLEI

Using a Fermi gas model of the nucleus Bethe[4] in 1937 showed that the nuclear level density ρ has the form

$$\rho \propto \exp(2\sqrt{aE}) \quad , \tag{1}$$

where E is the excitation energy of the nucleus, a is the nuclear level-density parameter, and the units are number of levels per MeV. The parameter a is given approximately by

$$\begin{aligned} a &= A/(\text{constant}) \\ &= A/8(\text{MeV}) \quad . \end{aligned} \tag{2}$$

Although much more accurate and sophisticated nuclear level-density calculations have been performed since Bethe's original work, these equations nevertheless give approximately the correct dependence upon excitation energy E and nuclear mass number A. Inspection shows that the heavier nuclei have, in general, the closest level spacings. For example, at an excitation energy of 3 MeV, the level density for a nucleus of mass A = 250 is a factor $e^{10.70} \cong 4.46 \times 10^4$ larger than that for a nucleus of mass A = 50. Clearly, closely spaced levels are to be found in heavy nuclei.

If one considers the nuclear shape degrees of freedom, it has been found that the level densities of deformed nuclei are generally greater than those of spherical nuclei with similar masses. This is due to the fact that in a deformed nucleus each intrinsic state gives rise to a rotational band, as one can easily see, for example, by coupling an odd particle to a rotating deformed even core.* The total level spectrum, for a fixed value of the total angular momentum I, is then obtained by <u>summing</u> over a set of intrinsic states rather than by a decomposition of the level spectrum, as in the case of a spherical system.[6] Thus,

$$\rho \text{ (deformed nuclei)} > \rho \text{ (spherical nuclei)}, \tag{3}$$

with all else fixed.

This enhancement is largest for a system that possesses full rotational degrees of freedom, that is, an equilibrium shape that has no rotational symmetry. In this case, the rotational band for a given intrinsic state involves (2I + 1) levels, and thus, for fixed I the level density enhancement is proportional to (2I + 1). If the equilibrium shape is invariant under a subgroup of rotations, the collective rotational degrees of freedom are correspondingly reduced, and the level density enhancement is not as much. In the case of axial symmetry, for example,[7] the enhancement is reduced by a factor corresponding to the width of the distribution of total angular momentum projected along the symmetry axis. The general result of these investigations[7] is that the enhancement in the nuclear level density of deformed nuclei is dependent upon the nuclear shape symmetry, which gives the number of collective rotational degrees of freedom. One therefore expects that the nuclei of closest level spacing are not only heavy nuclei, but also deformed nuclei.

Finally, an inspection of the first few MeV of excitation of nuclei that are well studied experimentally shows the following trends for the four types of nuclear species: the nuclear level density of discrete levels is smallest

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*The first calculations to describe the motion of a particle in a deformed well (core) were performed by Nilsson,[8] who constructed wave functions for such a system from basis states consisting of harmonic oscillator wave functions for each major shell. The total Hamiltonian in his calculations consists of the oscillator potential together with spin-orbit, ℓ^2, and deformed Y_{20} perturbation terms. The calculation was a major success in predicting ground state properties in regions of strong nuclear deformation.

for even-even nuclei, intermediate for even-odd and odd-even nuclei, and intermediate to large for odd-odd nuclei. Therefore, the nuclei of closest level spacing are most likely to be found among odd-odd, deformed, heavy nuclei.

IV. ISOMERISM IN NUCLEI

Nuclear isomeric levels are assumed to have the following four origins: differences in single-particle orbitals, shape and density isomers, level inversions due to the nucleon-nucleon interaction, and K selection rules. There may be other origins, such as pionic or quark degrees of freedom, which we shall not consider here.

Nuclear isomeric states are low-lying metastable states that commonly occur where the angular momentum differences between the metastable state and all lower states are large and where the corresponding energy differences are small. In these circumstances, electromagnetic transition probabilities T(EL) and T(ML) are stongly hindered because high-order multipoles are required and limited energy (frequency) is available. That is, approximately,

$$\left.\begin{matrix} T(EL) \\ \\ T(ML) \end{matrix}\right\} \propto (\Delta E)^{2L+1} \quad , \tag{4}$$

where ΔE is the (small) difference in excitation energy between the isomeric state and the final state, and where L is the (large) multipolarity of the transition. However, there are other mechanisms which can give rise to metastable states. In this section we consider four nuclear structure circumstances leading to isomerism.

A. Single-Particle Effects

The extreme single-particle shell model explains the existence of several regions or "islands" of isomerism that have been experimentally observed.[8] Namely, major shell closures at, for example, N,Z = 50, 82, or 126 are always preceded by nearly degenerate single-particle orbitals characterized by vastly different orbital and total angular momentum quantum numbers (ℓ,j). For example, Table II shows that the closed proton major shell at Z = 50, occurring for the filled orbital $1g_{9/2}$, is preceded by the $2p_{1/2}$ orbital. This gives orbital and total angular momentum values (ℓ,j) = (4,9/2) and (1,1/2), respectively. Similarly, the closed neutron major shell at N = 82 yields values

TABLE II

EXTREME SINGLE-PARTICLE SHELL-MODEL LEVELS

Protons

Level	n	Σn
$1s_{1/2}$	2	2
$1p_{3/2}$	4	6
$1p_{1/2}$	2	8
$1d_{5/2}$	6	14
$2s_{1/2}$	2	16
$1d_{3/2}$	4	20
$1f_{7/2}$	8	28
$2p_{3/2}$	4	32
$1f_{5/2}$	6	38
$2p_{1/2}$	2	40
$1g_{9/2}$	10	50
$1g_{7/2}$	8	58
$2d_{5/2}$	6	64
$1h_{11/2}$	12	76
$2d_{3/2}$	4	80
$3s_{1/2}$	2	82
$1h_{9/2}$	10	92
$2f_{7/2}$	8	100
$3p_{3/2}$	4	104
$2f_{5/2}$	6	110
$3p_{1/2}$	2	112
$1i_{13/2}$	14	126
$2g_{9/2}$	10	136

Neutrons

Level	n	Σn
$1s_{1/2}$	2	2
$1p_{3/2}$	4	6
$1p_{1/2}$	2	8
$1d_{5/2}$	6	14
$2s_{1/2}$	2	16
$1d_{3/2}$	4	20
$1f_{7/2}$	8	28
$2p_{3/2}$	4	32
$1f_{5/2}$	6	38
$2p_{1/2}$	2	40
$1g_{9/2}$	10	50
$2d_{5/2}$	6	56
$1g_{7/2}$	8	64
$3s_{1/2}$	2	66
$2d_{3/2}$	4	70
$1h_{11/2}$	12	82
$2f_{7/2}$	8	90
$1h_{9/2}$	10	100
$3p_{3/2}$	4	104
$2f_{5/2}$	6	110
$3p_{1/2}$	2	112
$1i_{13/2}$	14	126
$2g_{9/2}$	10	136
$3d_{5/2}$	6	142
$1i_{11/2}$	12	154
$2g_{7/2}$	8	162

$(\ell,j) = (5,11/2)$, $(2,3/2)$ for the $1h_{11/2}$ and $2d_{3/2}$ orbitals, respectively. That such pairs of orbitals are closely lying and the corresponding angular momentum differences are large yields precisely the conditions for isomerism, namely, small transition rates or long lifetimes, from Eq. (4). In this equation, the example of proton shell closure at $Z = 50$ gives $L = 4,5$, whereas the example of neutron shell closure at $N = 82$ gives $L = 4,5,6,7$. Thus, for small values of ΔE, in both examples, the values of the transition rates will be exceedingly small and the corresponding lifetimes exceedingly large, or isomeric.

B. Shape Isomers

When one calculates the total energy of a nucleus as a function of deformation, one finds in many nuclei that there is a second minimum, albeit with an energy greater than that corresponding to the ground state. This implies that an excited level of a nucleus might correspond to a quasi-bound state in the second well. These states may be isomeric because of the potential barrier that separates the two minima. Lifetimes of milliseconds have been observed.[9] Currently, the longest-lived shape isomer observed is in ^{242}Am with a lifetime of 25 ms. Because of the short lifetime and the difficulty in conceiving a viable method to utilize such states, it appears unlikely that shape isomers will provide a viable candidate.

C. Residual Interaction Isomers.

The third broad category of isomers occurs in many-particle systems and results from properties of the residual nucleon-nucleon system. Although the neutron-neutron and proton-proton interaction can on occasion give rise to inversion of levels (i.e., a sequence of levels for which the angular momentum is not monotonically increasing as the energy is increased), the most interesting cases arise from the neutron-proton (n-p) interaction.

The properties of the n-p interaction of interest were observed in the early days of the spherical shell model and were formalized into a set of rules by Nordheim.[10] The Nordheim rules specify which of several possible levels will be the ground state of odd-odd nuclei. These rules were then generalized to deformed nuclei in a straightforward fashion by Gallagher and Moszkowski.[11]

The physical origin of the rules is easy to see in a semi-classical picture. The nucleon-nucleon interaction is generally attractive and extremely short-ranged; in fact, a useful approximation often made is that it has zero-range. These two properties imply that the lowest lying two-nucleon state is that in which the spatial overlap is maximized. Further, as we know already from the deuteron, the nucleon-nucleon interaction is spin-dependent and is most attractive when the two nucleons are coupled to $S = 1$. (It may be useful to contrast this with the analogous case in atomic physics. The Coulomb force is repulsive, so the lowest two-electron states are those which minimize the spatial overlap. The rules formulated below will then be essentially the opposite of those in atomic physics.)

Figure 3 shows a nucleon moving in a nearly equatorial orbit having angular momentum j_a with z component m_a. Clearly, a second nucleon with angular momentum j_b can have maximum spatial overlap with the first nucleon if it is in an orbit with either $m_b = m_a$ or $m_b = -m_a$. (These statements are strictly valid only if $j_a = j_b$; however, one may show that they are still approximately true if $j_a \neq j_b$.)

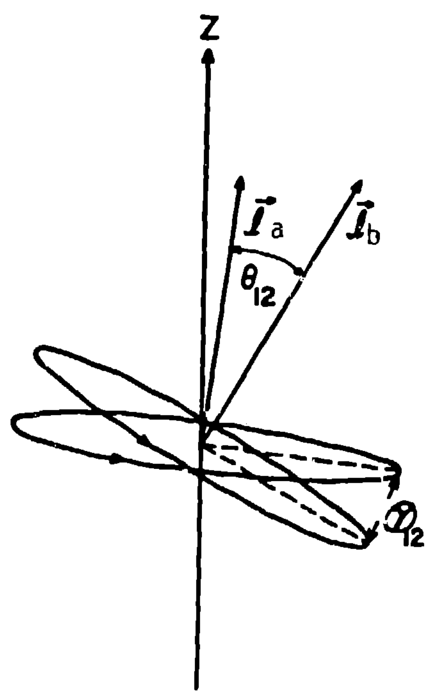

Fig. 3. Semi-classical picture of two nucleons moving in orbits so as to maximize their spatial overlap.

Suppose $j_a = j_b$; then the first case is not allowed by the Pauli principle if the two nucleons are identical. Thus, the lowest-lying two-proton or two-neutron state will be that in which $m_b = -m_a$. The superposition of all such states with the proper phase is just

$$\sum_m (-)^{j-m} |jm> |j-m> = \sqrt{2j+1}\ |j^2\ J=0> \quad .$$

Hence, the lowest state will have angular momentum zero, a result valid for <u>all</u> known even-even nuclei. For an n-p state, the corresponding state would have $|j_a - j_b|$.

If, however, the two nucleons are not identical, then one may also have the first case, namely, $m_a = m_b$. The lowest energy two-nucleon state can then have $J = j_a + j_b$. We must still decide if the lowest n-p state will be $J_{min} = |j_a - j_b|$ or $J_{max} = j_a + j_b$.

These properties are illustrated in Fig. 4 with two level schemes taken from Lederer et al.[2] In Fig. 4a is shown ^{42}Sc. In the simplest picture ^{42}Sc consists of a neutron and a proton outside a closed ^{40}Ca core. From Table II we note the nucleons will be in $f_{7/2}$ orbits. Thus, the lowest T = 0 levels will be either $J_{min} = 1$ or $J_{max} = 7$. In nature the two are nearly degenerate. (The ground state 0^+ has T = 1 and reflects the fact that the T = 1, S = 0 component of the residual interaction is stronger than the T = 0, S = 1 component. This may be viewed for our purposes as an anomaly because it is important only if the number of neutrons, N, is approximately the same as the number of protons, Z. In heavy nuclei N > Z.)

In Fig. 4b is shown the levels of ^{90}Zr that may be considered as two protons outside a closed ^{88}Sr core. Table II predicts that the two protons are in the $p_{1/2}$ and $g_{9/2}$ orbits. The $p_{1/2}^2$ and $g_{9/2}^2$ configurations have possible angular momentum and parity states of 0^+ and 0^+ through 8^+, which is consistent with the level diagram. The $p_{1/2}\ g_{9/2}$ configuration will give rise to states of $J_{min} = 4^-$ and $J_{max} = 5^-$. Our arguments above suggest that they should be close together but do not specify which should be lower. Note also that in Section 4a, the $p_{1/2}$, $g_{9/2}$ pair was predicted to give rise to isomers; indeed, the 5^- is an isomer with a lifetime of 809 ms. The 3^- level in ^{90}Zr is a collective octupole vibration that is not describable within our model space.

The Nordheim rules, which specify whether J_{min} or J_{max} is the lower state, are established by remembering that the neutron and proton prefer to couple to

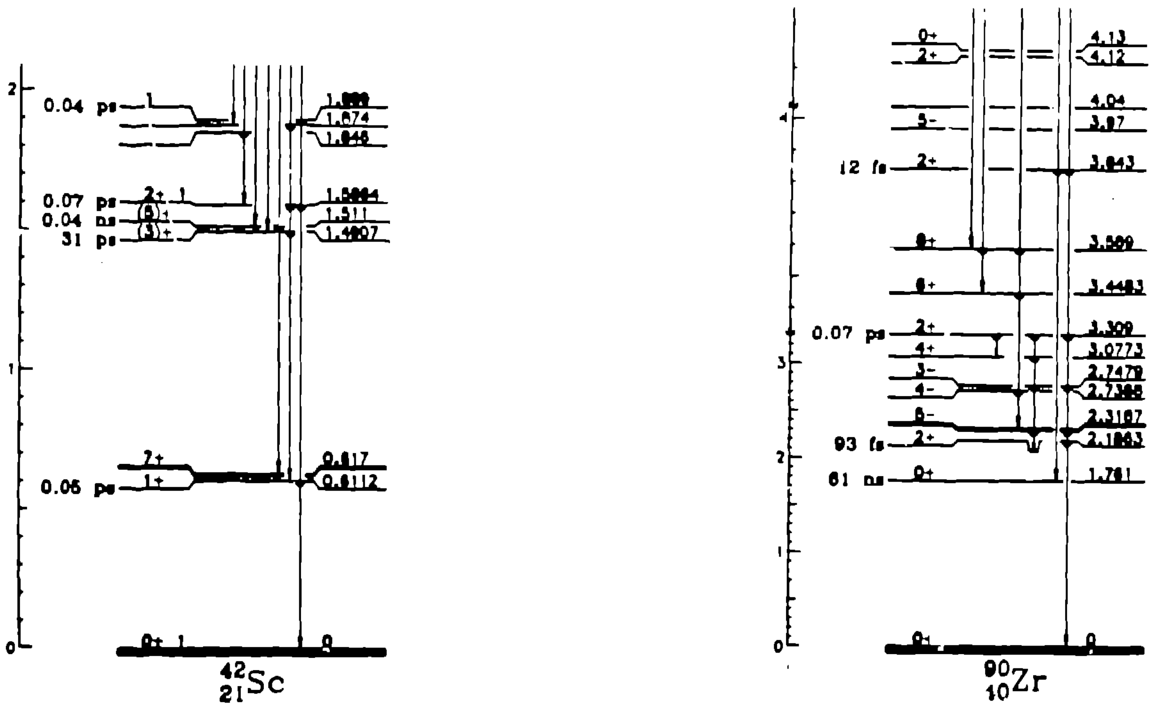

Fig. 4. Low-lying levels of (a) ^{42}Sc and (b) ^{90}Zr, which exhibit the influence of the neutron-photon interaction.

$S = 1$ as in the deuteron. Thus, we must write $\vec{J}$ in terms of the orbital angular momentum $\vec{\ell}$ and spin $\vec{s}$. We must then ensure that the two spins $\vec{s}_n$ and $\vec{s}_p$ point in the same direction so that $\vec{S} = \vec{s}_n + \vec{s}_p$ is necessarily one.

The arguments leading to the Nordheim rules are straightforward and we merely give the results. There are two distinct cases:

(1) $j_p = \ell_p \pm 1/2$, $\quad j_n = \ell_n \pm 1/2 \quad J = j_p + j_n$

(2) $j_p = \ell_p \pm 1/2$, $\quad j_n = \ell_n \mp 1/2 \quad J = |j_p - j_n|$.

The right-hand column gives the angular momentum of the lowest energy state. It must be stressed that these rules are empirical and may not hold in all cases (e.g., they fail for ^{42}Sc.) However, the Gallagher-Moskowski generalization is more reliable.

It is now clear how to form an isomer in a many-particle system: place a neutron in a single-particle orbit with $j_n = \ell_n \pm 1/2$ and a proton in an orbit with $j_p = \ell_p \pm 1/2$. If either or both of these single-particle orbits are not the ground state and have a single-particle angular momentum j greater than lower-lying single-particle orbits, an isomer will result. A good example is ^{118}Sb. The ground state is formed by a neutron in $d_{3/2}$ and a proton in $d_{5/2}$. The ground state spin is 1^+ as expected. One may excite the neutron to the $h_{11/2}$ orbit that lies 150 keV above the $d_{3/2}$. One would then expect a low-lying 8^- level, which is observed at 220 keV and has a lifetime of 5 hours.

Another example of isomers resulting from the strongly attractive n-p interaction is in ^{212}Po which consists of two neutrons and two protons outside a closed ^{208}Pb core. A proper calculation must allow the neutrons to occupy any of seven single-particle orbits and the protons of any six orbits. The resulting matrices are very large. However, if one uses the Kuo-Herling interaction,[12] which was obtained from the Hamada-Johnston[13] nucleon-nucleon interaction using many-body techniques,[14] one obtains the results of Fig. 5. There is a well-known isomer in ^{212}Po with a lifetime of 45 s of angular momentum 16 or 18. The theoretical calculation suggests it is 16^+, although the precision of the calculation is not sufficient to claim this unambiguously. The figure also gives an idea of the accuracy of current state-of-the-art, zero-parameter calculations.

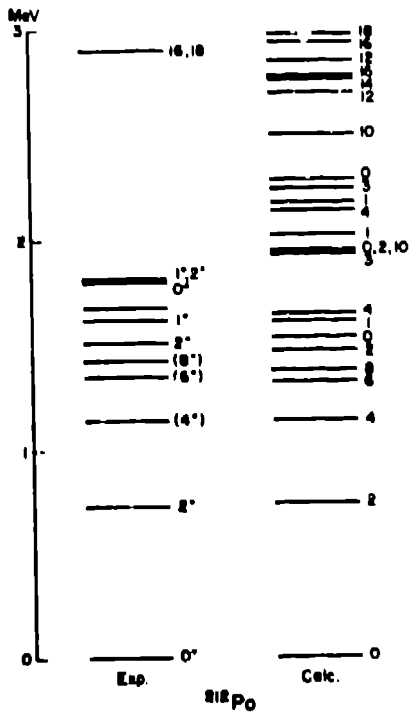

Fig. 5. Experimental and theoretical[15] (right column) levels of ^{212}Po. The two-body matrix elements used were those of Kuo and Herling.[12]

The conclusions from this section are that the n-p interaction can give rise to isomers, that a simplistic treatment of the n-p interaction is inadequate, and that there is some uncertainty in results obtained from zero-parameter calculations.

D. K-Selection Rules.

In this section we discuss a fourth method of obtaining isomers; by virtue of its method of construction it may hold the greatest promise of finding heretofore unknown isomers of use to a graser.

The wave function of an odd-A nucleon in the Bohr-Mottelson model[17] is written as

$$\Phi_I = \sqrt{\frac{2I+1}{16\pi^2}}\,[D^{I}_{MK}(\overset{*}{\theta})\chi_\Omega + (-)^{I+K}D^{I}_{M-K}(\theta)\chi_{-\Omega}] \qquad (5)$$

where the D^I_{MK} are rotation matrices describing the even-even core having angular momentum I. The projection of J upon the z-axis of the body-fixed axis is K. The function χ_Ω is a solution of the axially symmetric Nilsson Hamiltonian with Ω the projection of the single-particle angular momentum upon the body fixed axis. It is customary to use the notation $\Omega[Nn_Z\Lambda]$ to denote the states χ_Ω.

If the total angular momentum operator is $\vec{I} = \vec{R} + \vec{J}$ where $\vec{R}$ is the angular momentum operator of the even-even core, then the fact that the core is axially-symmetric implies that

$$R_Z\,\Phi = 0 = (I_Z - j_Z)\Phi = (K-\Omega)\Phi$$

or $K = \Omega$. Thus, we may speak interchangeably of K or Ω. The wavefunction (5) may now be labelled by I, M, and K. Although K is not a quantum number, it is an empirical fact that electromagnetic transitions between states of different K are hindered. The degree of hindrance varies, but a hindrance of an order 10 for each unit of K is a reasonable approximation.

Hence, in deformed nuclei, there is now another way to form long-lived states: one simply forms an excited state having a value of K much different from those which lie lower in energy. A good example is ^{178}Hf. The ground state of ^{178}Hf is constructed by putting the last two neutrons in the 7/2[514] orbit and the last two protons in 7/2[404]; the ground state will have K = 0. Suppose the proton pair is broken and one promotes one of the protons to 9/2[514]. The K values are z-components of angular momentum and simply add;

the K^{π} of the resultant state is 8^-. Two low-lying 8^- levels are known in ^{178}Hf; the lowest of these at 1.47 MeV has a halflife of 4 s, even though it could easily decay to either an 8^+ at 1.058 MeV or 6^+ at 0.632 MeV, each of which has K = 0. Obviously, a change in K of eight units is an effective hindrance of several orders of magnitude.

An even more interesting case is obtained by breaking both a neutron and proton pair. One then forms a state having K = 16. A level with angular momentum 16 is known experimentally at 2.447 MeV and has a lifetime of 31 years! It could decay by an E4 transition to a 12^+ at 2.15 MeV but the transition would require a change in K of 16 units. Thus, there is again experimental evidence for a large hindrance when the K number is violated.

The requirement for forming such isomers is the presence of high-Ω orbits near the Fermi level. There are a number of such single-particle orbitals in the rare earth and actinide region. Only a few such isomers are known but many more should exist. Experimental searches guided by theoretical suggestions have uncovered several isomers, but many more await discovery.

V. FIRST GUESS AS TO WHERE TO LOOK

Based upon the discussions of Secs. 3 and 4A, it is reasonable for an initial search to consider nuclei that are common to the set with close level spacings and the set displaying single-particle properties necessary for isomerism. To reiterate, the first set consists of odd Z-odd N, deformed, heavy nuclei and also, to a lesser extent, odd A, deformed, heavy nuclei. The second set consists of nuclei at or near closed major (or minor) shells such that low-lying single-particle states have large angular momentum differences compared with lower-lying states and the ground state. We note here that in regions of nuclear deformation we use single-particle quantum number appropriate to the Nilsson model[5] $\Omega[Nn_z\Lambda]$ instead of those appropriate to a single-particle moving in spherical potential well ($n\ell j$). Considering the union of these two sets, we find the following regions to be of interest in the search for graser candidate nuclei:

(a) odd Z-odd N rare earth nuclei ($150 \lesssim A \lesssim 190$)
(b) odd-A rare earth nuclei ($150 \lesssim A \lesssim 190$)
(c) odd Z-odd N actinide and trans-actinide nuclei A > 220
(d) odd-A actinide and trans-actinide nuclei A > 220
(e) odd Z-odd N nuclei with $39 \lesssim Z \lesssim 49$ and $57 \lesssim N \lesssim 65$
(f) odd-A nuclei with $39 \lesssim Z \lesssim 49$ and $57 \lesssim N \lesssim 65$.

VI. THEORETICAL NUCLEAR SPECTROSCOPY

In this section we shall briefly describe the main models used to calculate properties of nuclear levels, their accuracy and problems associated with each model. We shall attempt to keep the discussion of each model brief; there are many recent extensions, which, although providing more accuracy, only tend to obfuscate the main development. The models range from the purely phenomenological to microscopic; the former are fairly easy to use, but lack predictive power, while the latter are much more difficult and require enormous amounts of computer time. In this last category is Hartree-Fock, which is only now being developed for heavy nuclei. We refer the reader to the literature.[19]

A. Odd-A Nuclei.

Most descriptions of odd-A nuclei begin with the Bohr-Mottelson ansatz for the Hamiltonian:

$$H = \Sigma \frac{\hbar^2}{2\mathcal{I}_k} (I_k - j_k)^2 = \sum_k \frac{\hbar^2}{2\mathcal{I}_k} R_k^2 \ . \tag{6}$$

In Eq. (6) the $\mathcal{I}_k$ are the moments of inertia, and I_k, R_k and j_k are components of the total, core and single-particle angular momentum, respectively. We use the same notation as in Sec. 4. If two of the moments of inertia are identical, the system possesses axial symmetry. We begin by assuming there is no axial symmetry and then make the restriction later of $\mathcal{I}_x = \mathcal{I}_y$.

Expanding the quadratic in Eq. (6) and making some manipulations, one arrives at

$$H = \tfrac{1}{2}(A_1 + A_2)[\vec{I}^2 - I_3^2 + \vec{j}^2 - j_3^2] - 2(A_1 I_1 j_1 + A_2 I_2 j_2)$$
$$+ \tfrac{1}{2}(A_1 - A_2)(I_1^2 - I_2^2 + j_1^2 - j_2^2) + A_3(I_3 - j_3)^2 . \tag{7}$$

In Eq. (7) we have set

$$A_i = \frac{\hbar^2}{2\mathcal{I}_i} \quad \text{for convenience.}$$

Rewriting (7) in terms of raising and lowering operators gives

$$H = \tfrac{1}{2}(A_1 + A_2)(\vec{I}^2 - I_3^2 + \vec{j}^2 - j_3^2) - \tfrac{1}{2}(A_1 - A_2)(I_+ j_+ + I_- j_-)$$

$$- \frac{1}{2}(A_1 + A_2)(I_-j_+ + I_+j_-) + \frac{1}{4}(A_1 - A_2)(I_+^2 + I_-^2 + j_+^2 + j_-^2)$$

$$+ A_3(I_3 - j_3)^2 \quad . \tag{8}$$

If $\mathcal{I}_x = \mathcal{I}_y = \mathcal{I}$ or $A_1 = A_2$, Eq. (8) simplifies to

$$H = \frac{\hbar^2}{2\mathcal{I}} (\vec{I}^2 - I_3^2 + \vec{j}^2 - j_3^2 - I_-j_+ - I_+j_-) \quad . \tag{9}$$

The last term in (8) vanishes because $K = \Omega$, as proved in Section 4-D.

The Bohr-Mottelson wavefunction is just that of Eq. (5):

$$\Phi_{MK} = \sqrt{\frac{2I+1}{16\pi^2}} \, [D^I_{MK} \, \chi_K + (-)^{I-K} D_{M-K} \, \chi_{-K}] \tag{10}$$

with $\chi_K = \sum_j a_{jK} \, \phi_{jK}$,

ϕ_{jK}, the single-particle wavefunctions and the a_{jK} are given by the Nilsson model or some similar calculation. For an axially-asymmetric system, the wavefunction (10) must be summed over K.

The matrix elements of the raising and lowering operators are well-known:

$$\langle IK\pm1|I_{\mp}|IK\rangle = \sqrt{(I\pm K)(I\pm K+1)}$$

$$\langle j\Omega\pm1|j_{\pm}|j\Omega\rangle = \sqrt{(j\pm\Omega)(j\pm\Omega+1)}$$

and the matrix element of H may be readily calculated.

$$\langle IK|H|IK\rangle = \frac{\hbar^2}{2\mathcal{I}}(I(I+1) - K^2 + \langle IK|\vec{j}^2 - K^2|IK\rangle - \delta_{K\frac{1}{2}}(-)^{I+\frac{1}{2}}(I+\tfrac{1}{2})a) \tag{11}$$

The matrix element of $\vec{j}^2 - j_3^2$ is usually called the recoil term. The last term in Eq. (11) arises from the last two operators of Eq. (9) which are the Coriolis interactions. The Coriolis force arises from a cross matrix element connecting K and -K and hence is only non-zero if $K = \frac{1}{2}$. The quantity a is called the decoupling parameter and is equal to

$$a = \sum_j (-)^{j-\frac{1}{2}} (j+\tfrac{1}{2})(a_{j\frac{1}{2}})^2 \quad . \tag{12}$$

The $a_{j\frac{1}{2}}$ are just the expansion coefficients of a Nilsson wavefunction. Expression (11) gives the diagonal contribution to the energy of odd-A nuclei. The recoil term is usually dropped, albeit without justification. The moment of inertia may be obtained either from the neighboring even-even nucleus or by fitting. In fact, a code NUCLEV has been constructed*, which completes the low-level ($E < 3$-5 MeV) spectrum of an odd-A deformed nucleus given the spin, parity, and excitation energies of two members of each band for $K \neq 1/2$.

There may also be off-diagonal matrix elements arising from the Coriolis interations:

$$\langle IK'|H|IK\rangle = -A \sqrt{(I \pm K)(I \pm K+1)}\, \langle \chi_{K'}|j_{\pm}|\chi_K\rangle\, \delta_{K',K+1} \quad . \tag{13}$$

Such terms will give rise to mixing of different K bands and may also alter the energy dependence from that of Eq. (11). It has been observed for some time that mixing predicted by Eq. (13) is too large[17] and must be arbitrarily reduced by 30-40%. The origin of this attenuation factor is only beginning to be understood and must still be considered necessary but arbitrary.

A second arbitrary approximation made is to drop the recoil term, usually arguing it can be absorbed into the central, single-particle potential field. However, if the odd nucleon is allowed to occupy two or more single-particle orbits, this is clearly not allowed since H_{recoil} depends on K. Also, if there are several valence nucleons--and these will be the cases of greatest interest --then

$$\begin{aligned} H_{recoil} &= A(\vec{J}^2 - j_3^2) \\ &= A(\sum_i \vec{j}^2(i) - (\Sigma\, \vec{j}(i))^2 + 2\Sigma\, \vec{j}(i) \cdot \vec{j}(k)) \quad . \end{aligned}$$

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*D. G. Madland, private communication, October 1982.

Thus, H_{recoil} contains one-body as well as two-body operators and cannot be put into the single-particle potential. Further, if the pairing interaction is taken into account, then even if there is only one valence nucleon, H_{recoil} will be effectively a many-body operator.[18]

In general, a careful calculation leads to agreement with experiment to better than 20 keV and often better, although certain nuclei provide examples where agreement is not as good. An example is ^{163}Er. However, most of the deviations can be understood in terms of K-band mixing or other nearby single-particle orbits that must be included; other examples appear to defy rational explanation. In certain nuclei around mass 180 axial asymmetry is known to be important.[16] Then K and Ω are no longer identical and the full Hamiltonian Eq. (8) must be used.

B. Odd-Odd Nuclei.

In the simplest generalization of (9) to include two extra nucleons, one has

$$H = \sum_{\kappa} \frac{\hbar^2}{2\mathcal{I}_{\kappa}} (I_{\kappa} - j^n_{\kappa} - j^p_{\kappa})^2 + V_{np} \quad , \tag{14}$$

with V_{np}, the residual neutron-proton interaction.

Only the axial case has been worked out in detail, there having been sufficient uncertainty with the n-p interaction and experimental level schemes that the generalization to complete asymmetry was unwarranted.

Expanding (14) leads to terms that are similar to those encountered in the odd-A case. With the wave function

$$\Phi^I_{mK} = \sqrt{\frac{2I+1}{16\pi^2}} \, [D^I_{mK} \, \chi_{\Omega p} \, \chi_{\Omega n} + (-)^{K+I} \, D^I_{m-K} \chi_{-\Omega p} \chi_{-\Omega n}] \quad , \tag{15}$$

the only term in H new is the residual n-p interaction. Using an argument identical to that which proved $K = \Omega$ in odd-A nuclei, here one may easily demonstrate that $K = \Omega_n + \Omega_p$. The matrix element of V_{np} is

$$\begin{aligned} \langle IK|V_{np}|IK'\rangle &= \langle \Omega_p \, \Omega_n | V_{np} | \Omega'_p \Omega'_n \rangle \\ &+ (-)^{K+I} \, \delta_{K-K'} \, \langle \Omega_p \Omega_n | V_{np} | -\Omega'_p \, -\Omega'_n \rangle \quad . \end{aligned} \tag{16}$$

The first term on the right-hand side is called the Gallagher-Moszkowski[11] matrix element; the second the Newby[20] matrix element. Because V_{np} must be rotationally invariant, one must have $\Omega = \Omega_p + \Omega_n$ equal in both the bra and ket. In particular, this implies that the Newby matrix element will only contribute to K = 0 bands.

The matrix element (16) may be rewritten as

$$\langle IK|V_{np}|\ IK\rangle = A + (-)^I\ \delta_{K,0}B \tag{16}$$

where A and B are constants for a given set of single-particle oribtals, $\chi_{\Omega n}$ and $\chi_{\Omega p}$. The values of A are on the order magnitude of 100 keV, although there is a considerable variation. The largest value of A so far observed is in ^{188}Re, for which A is 330 keV. The values of B obtained empirically are smaller than the values of A and are around 50 keV. Even this value produces a dramatic and unique effect on the spectrum since the phase $(-)^I$ gives rise to an alternating shift in the energies of members of ground state bands. The errors in the values of A and B are 10 to 20 keV.

The moment of inertia of odd-odd nuclei, I_{00}, will, in general, differ from that in neighboring nuclei. Partially successful attempts have been made to calculate I_{00}. The common procedure is to use the Takahashi rule, which relates I_{00} to the moments of inertia of neighboring nuclei:

$$I_{00} = I_p + I_n - I_{ee} \quad . \tag{17}$$

In Eq. (17) I_p and I_n are the moments of inertia of the neighboring odd-A nucleus having an odd proton and neutron, respectively, and I_{ee} is the moment of inertia of the even-even nucleus. This procedure has proven very successful. Perhaps the most exhaustive test of this procedure is in the work of Hoff and collaborators[21] who are able to accurately reproduce the positions of large numbers of low-lying levels. For example, in ^{176}Lu good agreement is obtained for members of the 13 lowest bands.

VII DISCUSSION

We have in this paper attempted to give a brief overview of some aspects of nuclear structure relevant to the search for gamma-ray lasers. It seems to us that the two crucial steps in developing a gamma-ray laser are the identification of a mechanism to allow lasing and the identification of a nuclear isomer with the appropriate lifetime and properties that would allow its use as a storage level. We have addressed the latter issue.

In particular, we have summarized the current knowledge of known long-lived isomers that have nearby levels. Within existing experimental limitations, no nuclear isomer meets all of the requirements imposed by the present scenarios. Assuming that one must search for additional isomers, which may have better prospects of having the desired properties, we suggested those regions of the nuclear mass table that would be most suitable to search first. We have tried to convey our belief that there exist many isomers yet undiscovered.

Because the experimental searches will be both costly and time consuming, it would appear prudent to initially seek theoretical guidance as to where to initiate a search. We therefore summarized some of the current models that can be used to calculate nuclear levels. The accuracy of the models is on the order of 10 keV, although in some circumstances it can be less. Although the accuracy is not sufficient to identify <u>the</u> candidate, it is sufficient to eliminate many would-be candidates. In addition, current work promises to reduce the numerical uncertainty further. If the search for a gamma-ray laser is to be successful, it must necessarily be a result of a collaboration between, not only atomic and nuclear physicists, but also experiment and theory.

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